复习：

- 力法应用（简单刚架、简单板架）

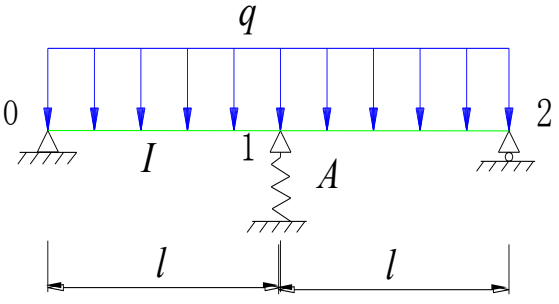
本堂课主要内容

- 五弯矩方程
- 阶梯变断面梁的计算
- 甲板板架计算

§4-3 弹性支座上连梁的计算

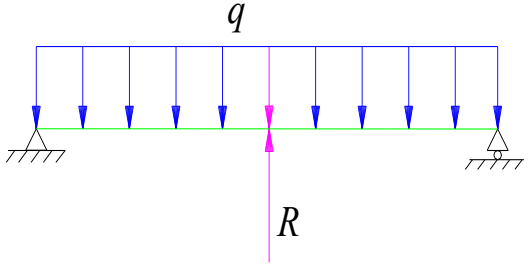
1、五弯矩方程

例：已知：$E, I, l, q, A = \dfrac{l^3}{6EI}$

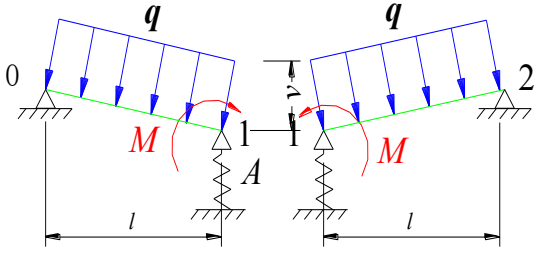


分析：

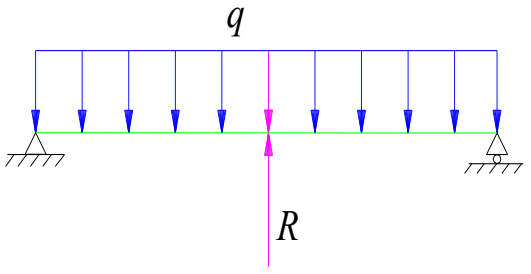
方法1：中间支座去掉代支反力R



方法2：中间支座切开代弯矩M_1



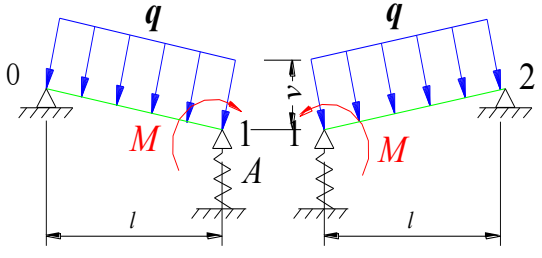
方法1：



- 去掉中间弹性支座代以支反力 R ，
- 列变形连续方程：

$$v_1 = v_{1支} \Rightarrow \frac{5q\cdot(2l)^4}{384EI} - \frac{R(2l)^3}{48EI} = AR = \frac{Rl^3}{6EI} \Rightarrow R = \frac{5}{8}ql$$

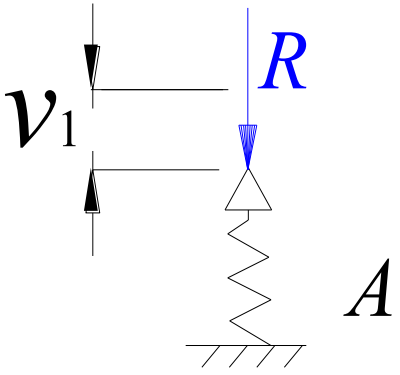
方法2：



- 中间支座断面切开，代以弯矩 M_1 ，
- 列支座断面转角连续方程：

$$\theta_{10} = \theta_{12} \Rightarrow \frac{Ml}{3EI} - \frac{ql^3}{24EI} + \frac{v_1}{l} = -\frac{Ml}{3EI} + \frac{ql^3}{24EI} - \frac{v_1}{l} \quad \cdots\cdots\cdots(1)$$

列补充方程：



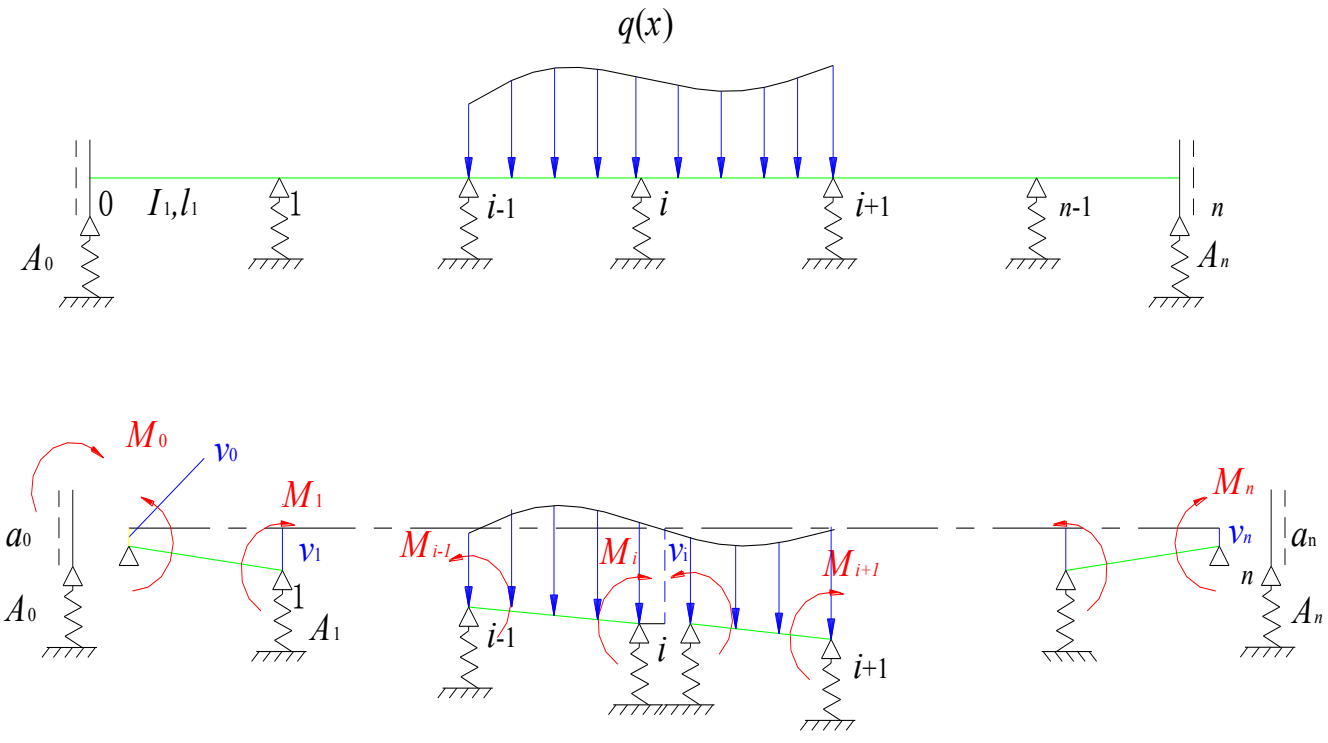
支座1：$v_1 = AR_1$

$$R_1 = R_1' + R_1'' = \frac{2(M_1 + \frac{1}{2}ql^2)}{l} = 2\left(\frac{M_1}{l} + \frac{ql}{2}\right)$$

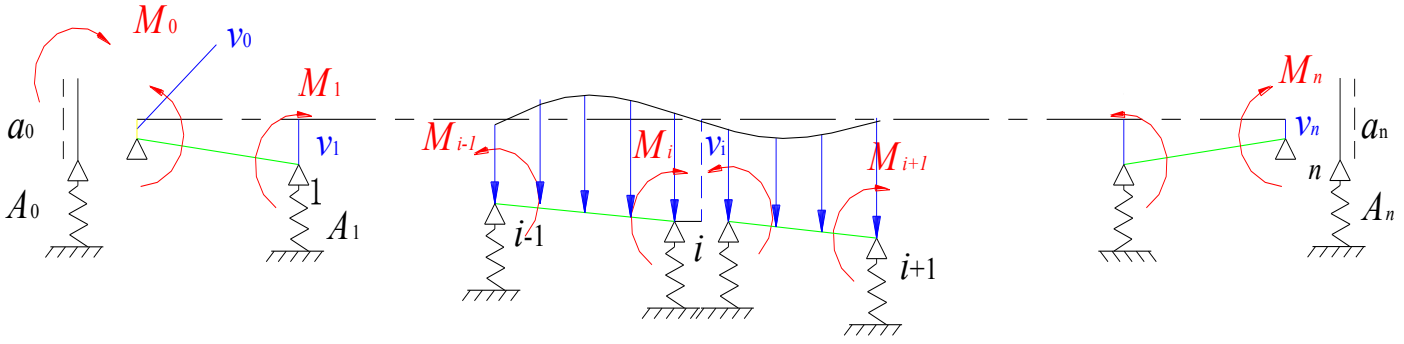
$$v_1 = AR_1 = \frac{l^3}{3EI}\left(\frac{M_1}{l} + \frac{1}{2}ql\right).....(2)$$

（1）、（2）联立解得：$M = -\frac{3}{16}ql^2$

推广：



- 在支座断面切开，得到n+1个未知弯矩 $M_i(i=0,1,2,.....n)$
- **列出各支座处的转角连续方程：** $\left(\theta_{i左}=\theta_{i右}\right)$



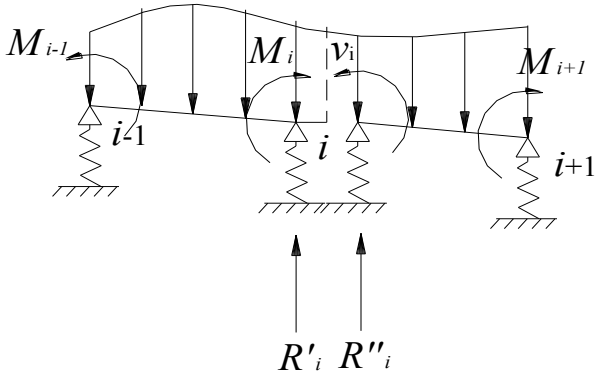
O支座：$\alpha_0 M_0 = \theta_{10} \quad \Rightarrow \alpha_0 M_0 = -\dfrac{M_0 l_1}{3EI_1} - \dfrac{M_1 l_1}{6EI_1} + \theta_0(q_1) + \dfrac{v_1 - v_0}{l_1}$

i支座：$\theta_{ii-1} = \theta_{ii+1}$

$$\Rightarrow \frac{M_{i-1} l_i}{6EI_i} + \frac{M_i l_i}{3EI_i} + \theta_i(q_i) + \frac{v_i - v_{i-1}}{l_i} = \frac{M_i l_{i+1}}{3EI_{i+1}} - \frac{M_{i+1} l_{i+1}}{6EI_{i+1}} + \theta_i(q_{i+1}) + \frac{v_{i+1} - v_i}{l_{i+1}}$$

n支座：

$$\theta_{nn-1} = \theta_n \Rightarrow \frac{M_{n-1} l_n}{6EI_n} + \frac{M_n l_n}{3EI_n} + \theta_n(q_n) + \frac{v_n - v_{n-1}}{l_n} = -\alpha_n M_n$$



- **列补充方程：**

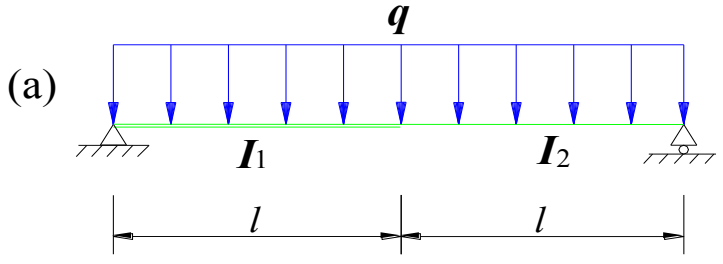
$$v_i = A_i R_i = A_i \left[\frac{M_i - M_{i-1}}{l_i} + r_i(q_i) + \frac{M_i - M_{i+1}}{l_{i+1}} + r_i(q_{i+1}) \right] \ldots i = 1, 2, \cdots$$

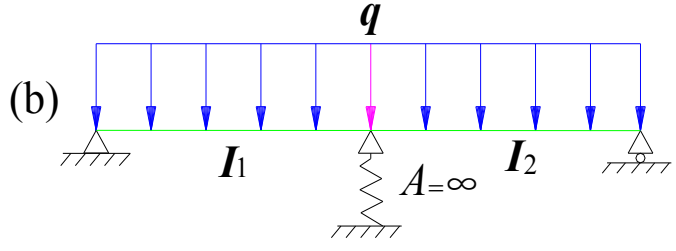
$\theta_i(q_i)$ ——第 i跨上的外载荷在支座处引起的转角；

$r_i(q_i)$——第 i跨处的外载荷在支座处引起的支反力。

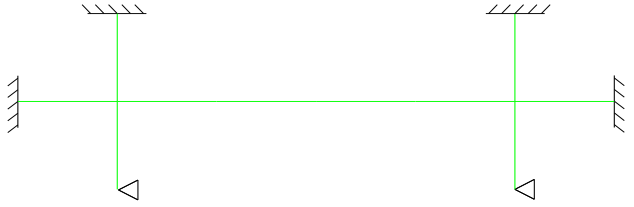
- **整理得：五弯矩方程**
- **作弯矩、剪力图**

2、阶梯形变断面梁的计算



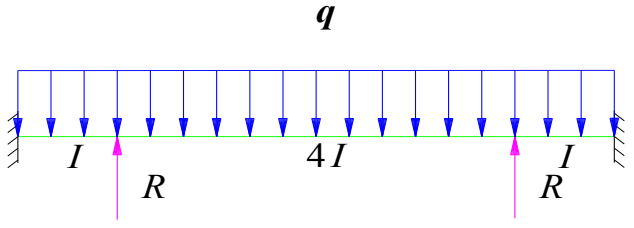


3、甲板板架的计算

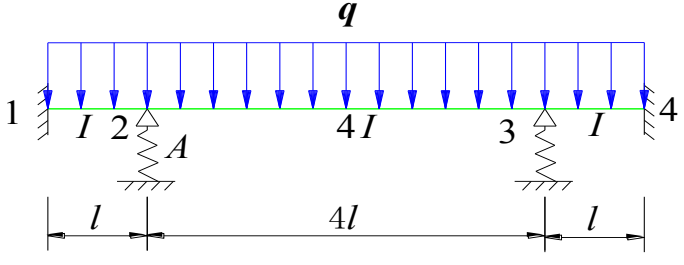


解题步骤：

1、计算舱口端横梁作为甲板纵桁弹性支座的柔性系数 A ；



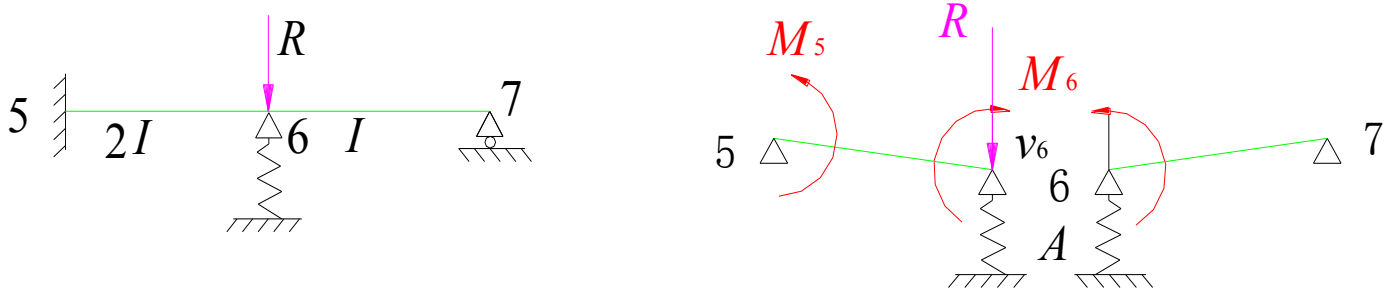
2、用五弯矩方程计算甲板纵桁；



3、再计算舱口端横梁

1、计算舱口端横梁作为甲板纵桁弹性支座的柔性系数

（1）、虚设弹支 $(A=\infty)$，支座5、6切开代以未知 M_5, M_6



（2）、变形连续方程：

$$\theta_{56}=0\Rightarrow-\frac{M_5 l}{3E(2I)}-\frac{M_6 l}{6E(2I)}+\frac{v_6}{l}=0\ldots(a)$$

$$\theta_{65}=\theta_{67}\Rightarrow\frac{M_5 l}{6E(2I)}+\frac{M_6 l}{3E(2I)}+\frac{v_6}{l}=-\frac{M_6 l}{3EI}-\frac{v_6}{l}\ldots.(b)$$

（3）、列补充方程：　　$v_6 = AR_6, \because$　　$\therefore R_6 = 0$

$$R_6 = R_6' + R_6'' = \left(\frac{M_6 - M_5}{l} + R \right) + \frac{M_6}{l} = \frac{2M_6 - M_5}{l} + R = 0 \ldots . (c)$$

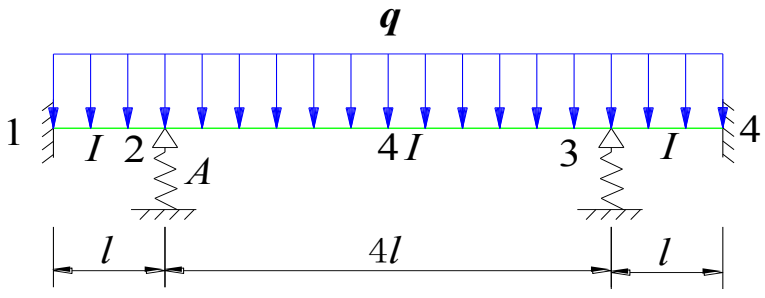
(4)、联立（a）(b)(c)解得：

$$\begin{cases} M_5 = \dfrac{4}{9} Rl = 0.445Rl \\ M_6 = \dfrac{5}{18} Rl = -0.278Rl \\ v_6 = \dfrac{11}{216} \dfrac{Rl^3}{EI} = 0.0509 \dfrac{Rl^3}{EI} \end{cases}$$

- **舱口端横梁作为甲板纵桁的弹性支座的柔性系数**

$$A = v_6 \Big|_{R=1} = 0.0509 \frac{l^3}{EI}$$

2、用五弯矩方程计算甲板纵桁



（1）、应用对称简化，未知数 ：

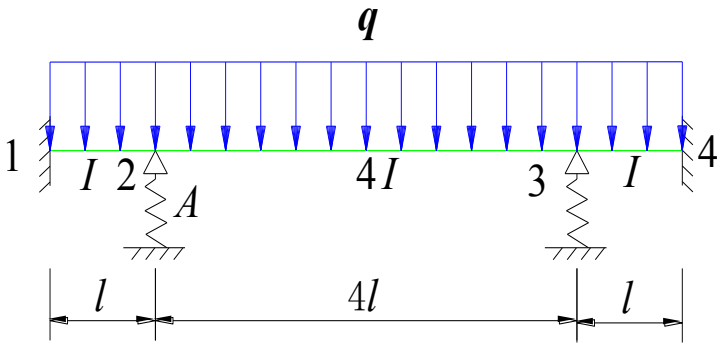
$$M_1 = M_4, M_2 = M_3, v_2 = v_3 = v$$

（2）、转角连续方程：

$$\theta_{12} = 0 \Rightarrow -\frac{M_1 l}{3EI} - \frac{M_2 l}{6EI} + \frac{q l^3}{24EI} + \frac{v}{l} = 0 \ldots\ldots (d)$$

$$\theta_{21} = \theta_{23} \Rightarrow \frac{M_1 l}{6EI} + \frac{M_2 l}{3EI} - \frac{q l^3}{24EI} + \frac{v}{l} = -\frac{M_2 (4l)}{3E(4I)} - \frac{M_2 (4l)}{6E(4I)} + \frac{q (4l)^3}{24E(4I)} \ldots\ldots (e)$$

（3）、列补充方程：



支座2： $v = AR = A\left(R_2' + R_2''\right)$

$$\sum m_1 = 0 \Rightarrow R_2' = \frac{M_2 - M_1}{l} + \frac{ql}{2}$$

$$\sum m_3 = 0 \Rightarrow R_2'' = \frac{M_2 - M_3}{4l} + \frac{1}{2}\left(q \times 4l\right) = 2ql$$

$$\therefore v = AR = 0.0509\frac{l^3}{EI}\left(\frac{M_2 - M_1}{l}\right) + \frac{5}{2}ql.....(f)$$

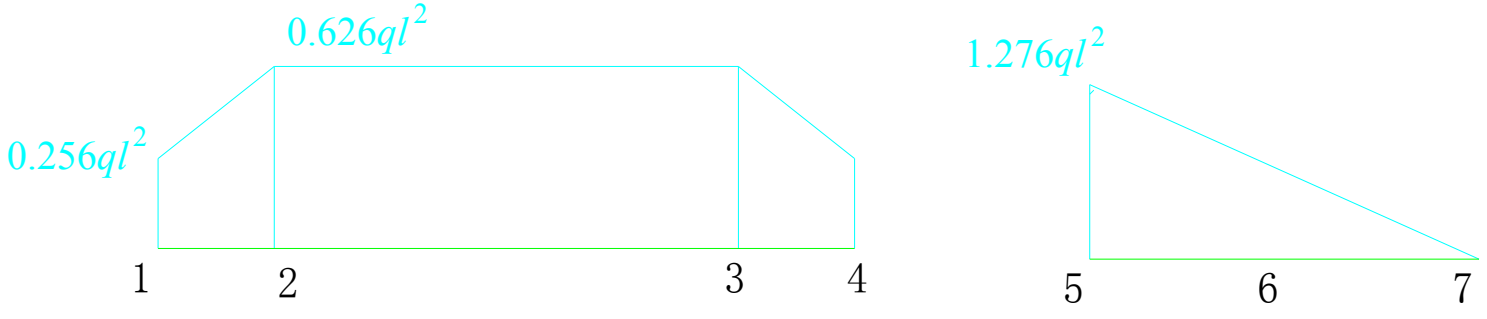
（4）、联立（d）（e）（f）解得：

$$M_1 = 0.256ql^2,\ \ M_2 = 0.626ql^2,\ \ v = 0.146\frac{ql^4}{EI}\left(\text{弹支位移}\right)$$

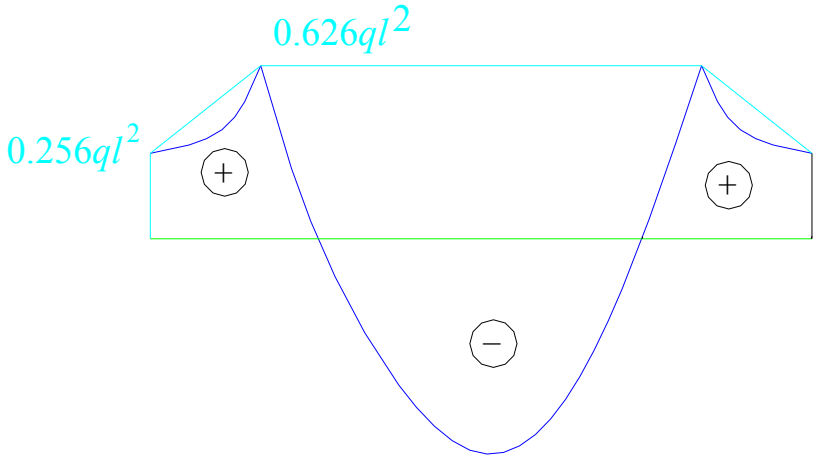
3、再计算舱口端横梁（代入第一步结果）

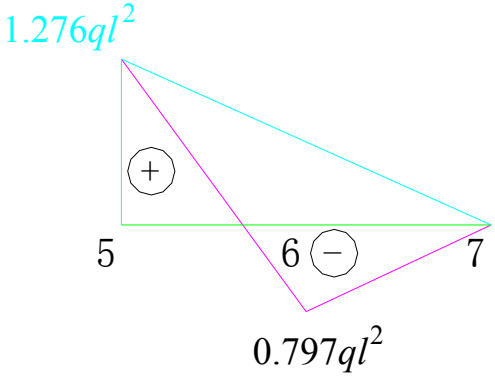
$$M_5 = 0.445Rl = 1.276ql^2,\quad M_6 = -0.278Rl = -0.797ql^2$$

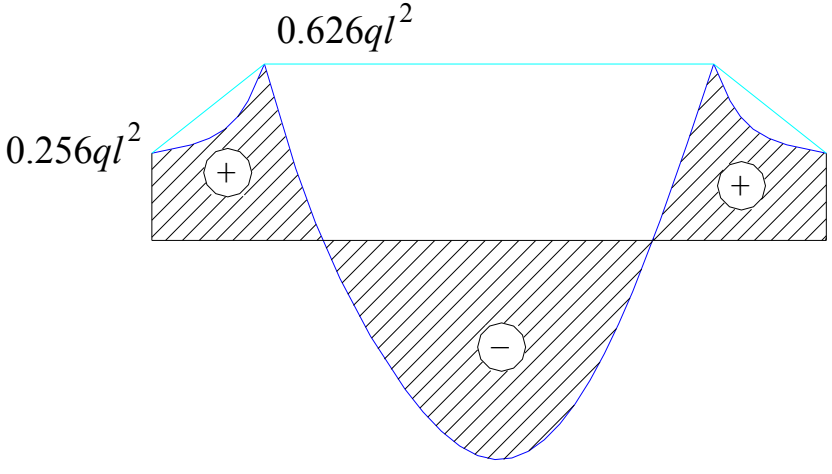
4、画*M*图：

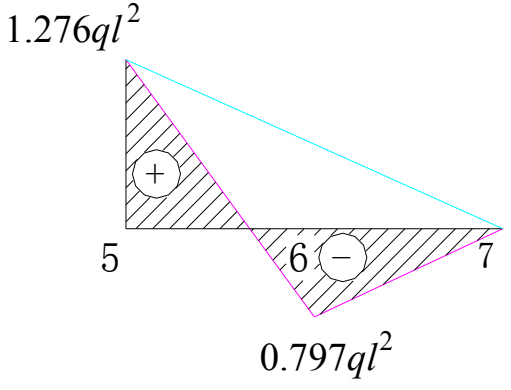


M图


0.626ql²
0.256ql²
+
+
−


1.276ql²
+
5
6
−
7
0.797ql²


0.626ql²
0.256ql²
+
+
−
1.374ql²


1.276ql²
+
5
6
−
7
0.797ql²

本堂小结：

1) 五弯矩方程及其应用；

2）用力法求解弹性支座上连续梁，列转角连续方程时需要考虑弹支位移引起的转角。另需列补充方程：$v = AR(R = f(M))$

3）阶梯变断面梁可在突变处虚设弹性支座 $(A = \infty)$ ，列补充方程 $R = 0$

4）三弯矩、五弯矩方程为力法的主要方法。

- 作业：4.8、4.9、4.10