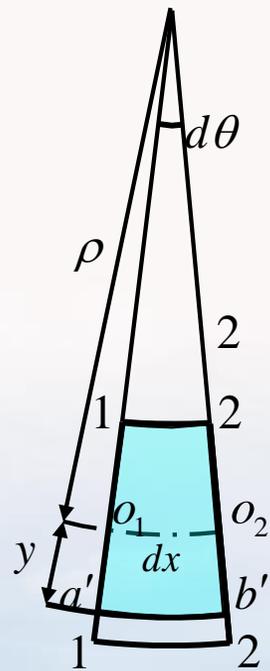
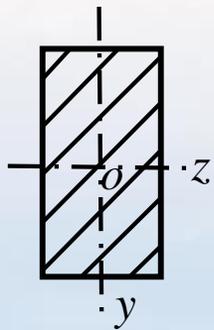
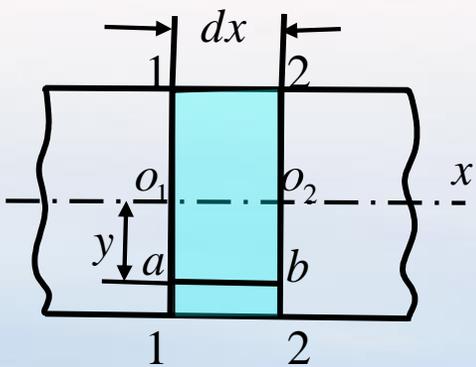


弯曲正应力



梁的应力

1. 几何关系



$$\begin{aligned}\varepsilon &= \frac{a'b' - \overline{ab}}{\overline{ab}} \\ &= \frac{(\rho + y) \cdot d\theta - dx}{dx} \\ &= \frac{(\rho + y) \cdot d\theta - \rho d\theta}{\rho d\theta}\end{aligned}$$

$$= \frac{y}{\rho}$$

$$\varepsilon = \frac{y}{\rho} \quad (1)$$

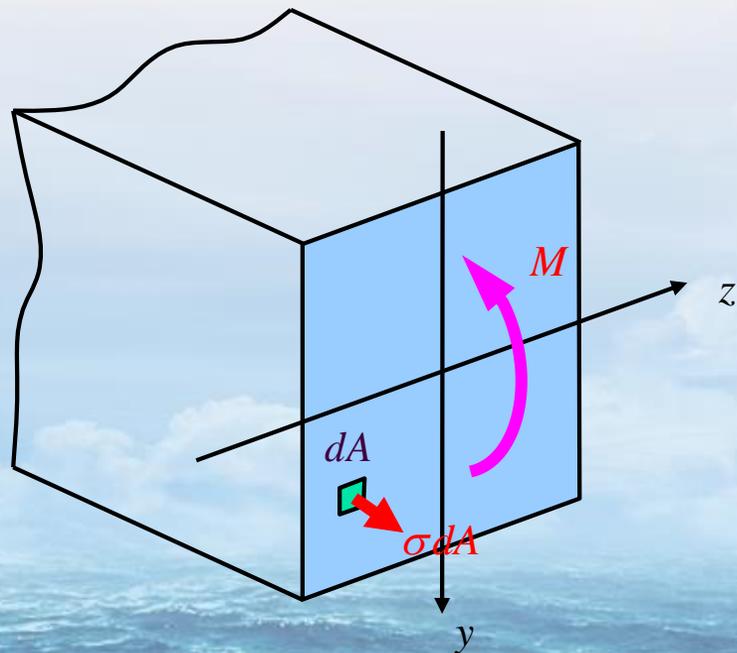


梁的应力

2. 物理关系

$$\sigma = E\varepsilon \xrightarrow{\varepsilon = \frac{y}{\rho}} \sigma = E \frac{y}{\rho} \quad (2)$$

3. 静力学关系



$$N = \int_A \sigma \cdot dA = 0 \quad (3)$$

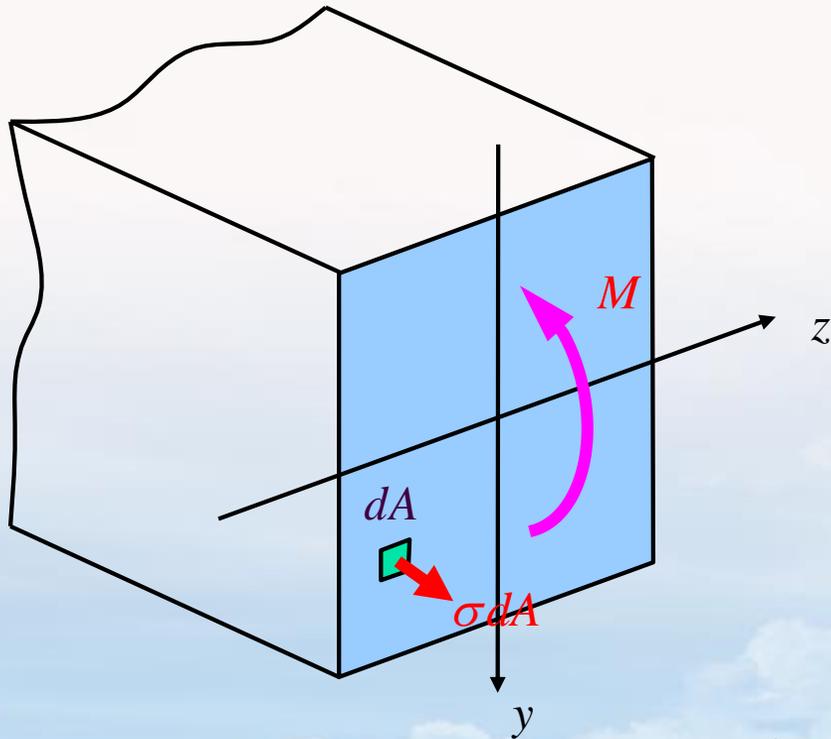
$$M_y = \int_A z \cdot \sigma \cdot dA = 0 \quad (4)$$

$$M_z = \int_A y \cdot \sigma \cdot dA = M \quad (5)$$



梁的应力

$$N = \int_A \sigma \cdot dA = 0 \quad (3)$$



$$\int_A \sigma \cdot dA = 0$$

↓ $\sigma = E \frac{y}{\rho}$

$$\int_A E \frac{y}{\rho} \cdot dA = 0$$

↓

$$\int_A y \cdot dA = 0$$

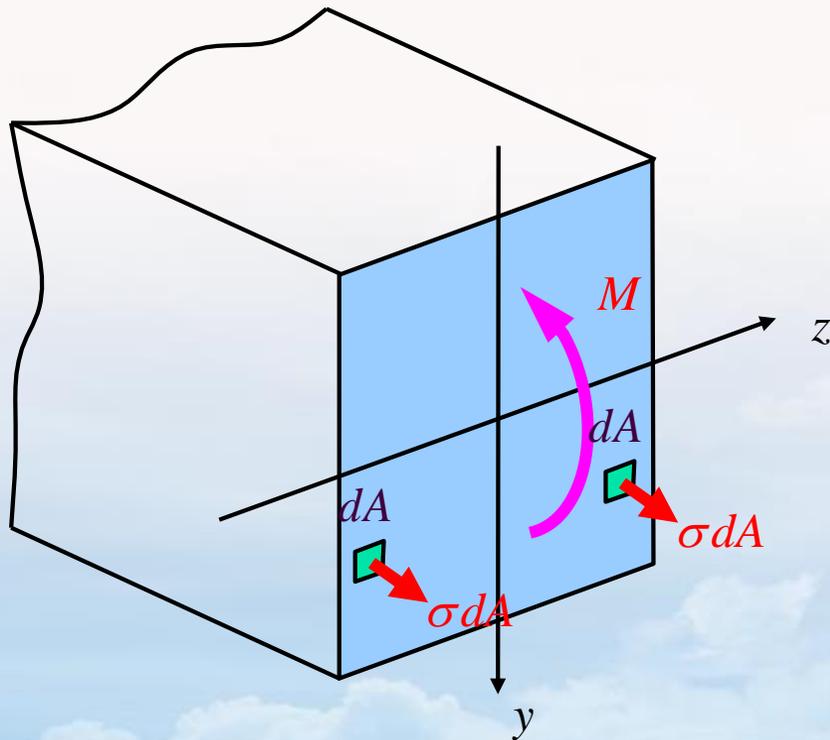
$$y_c \cdot A = \int_A y \cdot dA = 0 \longrightarrow y_c = 0$$

Z轴通过横截面的形心



梁的应力

$$M_y = \int_A z \cdot \sigma \cdot dA = 0 \quad (4)$$



$$\int_A z \cdot \sigma \cdot dA = 0$$

↓ $\sigma = E \frac{y}{\rho}$

$$\int_A z \cdot E \frac{y}{\rho} \cdot dA = 0$$

↓

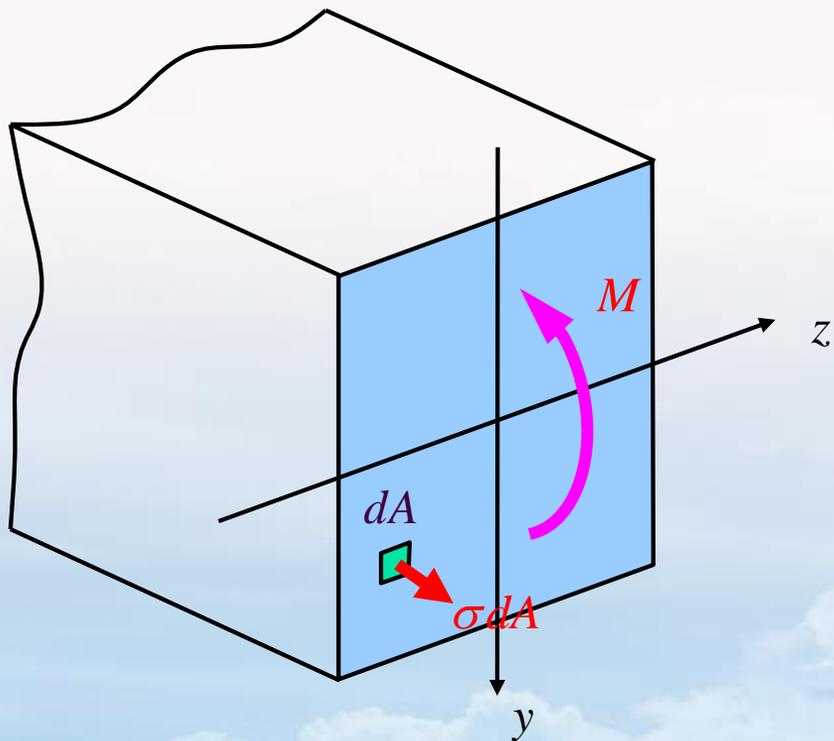
$$\int_A z \cdot y \cdot dA = 0$$

自然满足



梁的应力

$$M_z = \int_A y \cdot \sigma \cdot dA = M$$



$$\int_A \sigma \cdot y \cdot dA = M$$

↓ $\sigma = E \frac{y}{\rho}$

$$\int_A E \frac{y}{\rho} \cdot y \cdot dA = M$$



$$\frac{E}{\rho} \int_A y^2 \cdot dA = M$$

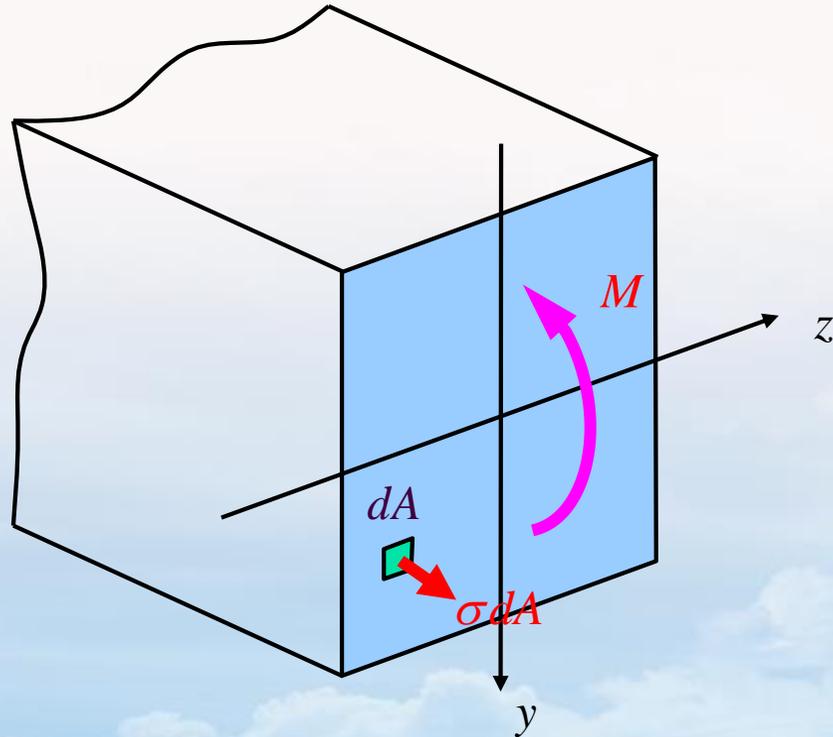
J_z : 截面对z轴的惯矩

$$\frac{1}{\rho} = \frac{M}{EJ_z}$$

①



梁的应力



$$\frac{1}{\rho} = \frac{M}{EJ_z} \quad (1)$$

EJ_z : 抗弯刚度



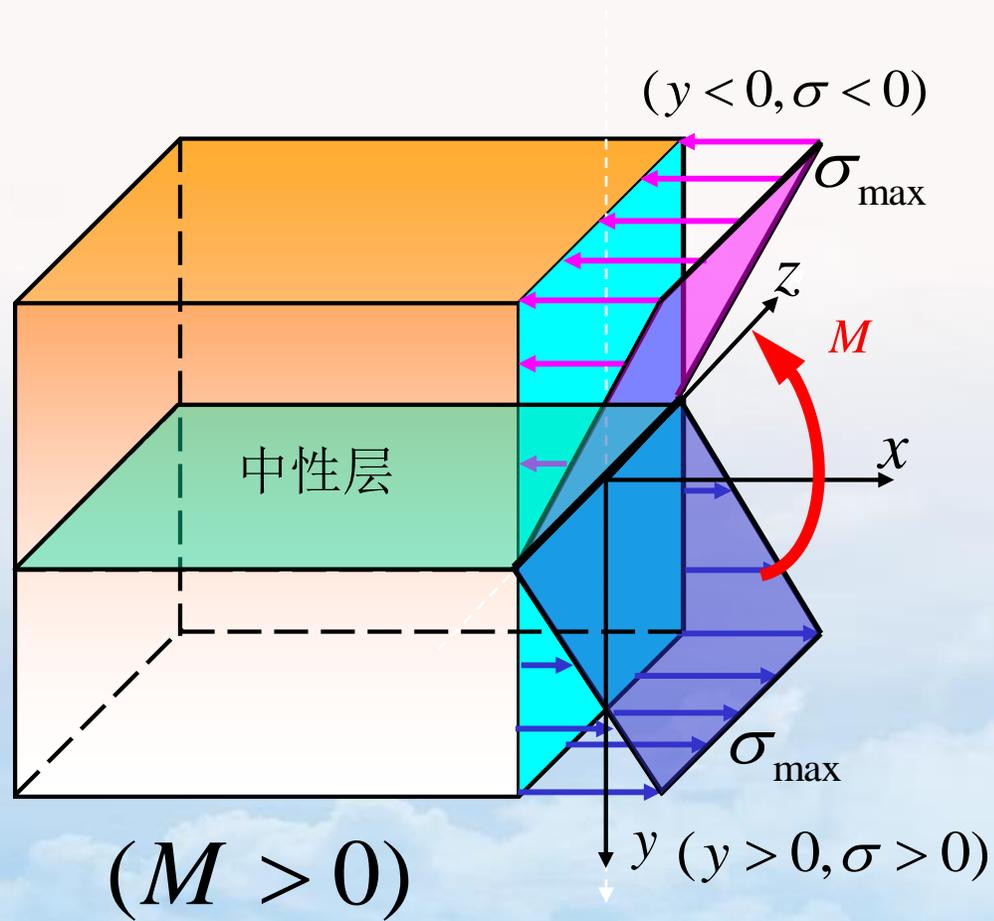
$$\sigma = E \frac{y}{\rho}$$

$$\sigma = \frac{M}{J_z} y$$

(2)



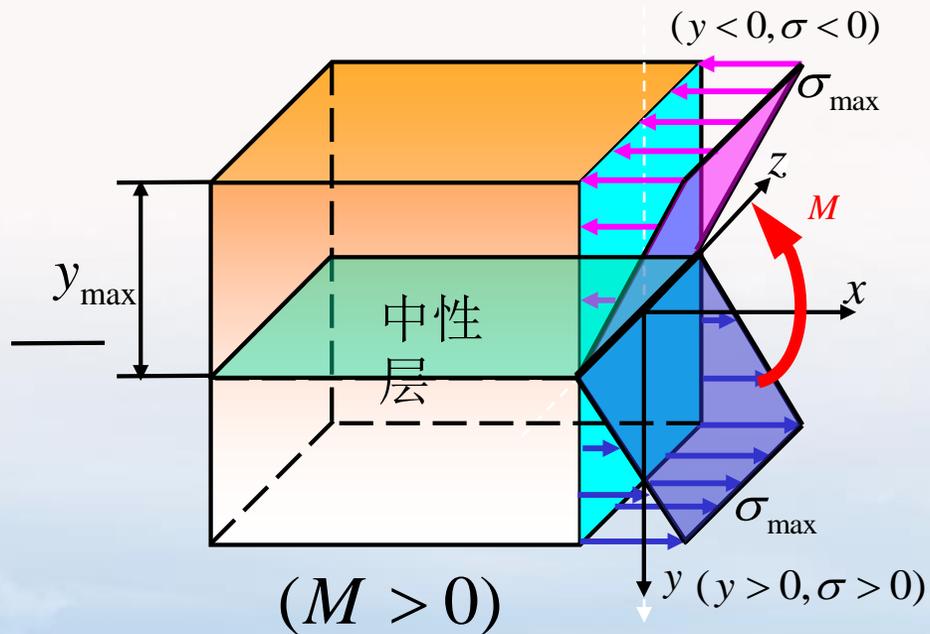
梁的应力



$$\sigma = \frac{M}{J_z} y$$



梁的应力



$$\sigma = \frac{M}{J_z} y \quad (2)$$

当 $y = y_{\max}$ 时, 则 $\sigma = \sigma_{\max}$

$$\sigma_{\max} = \frac{M}{J_z} y_{\max}$$



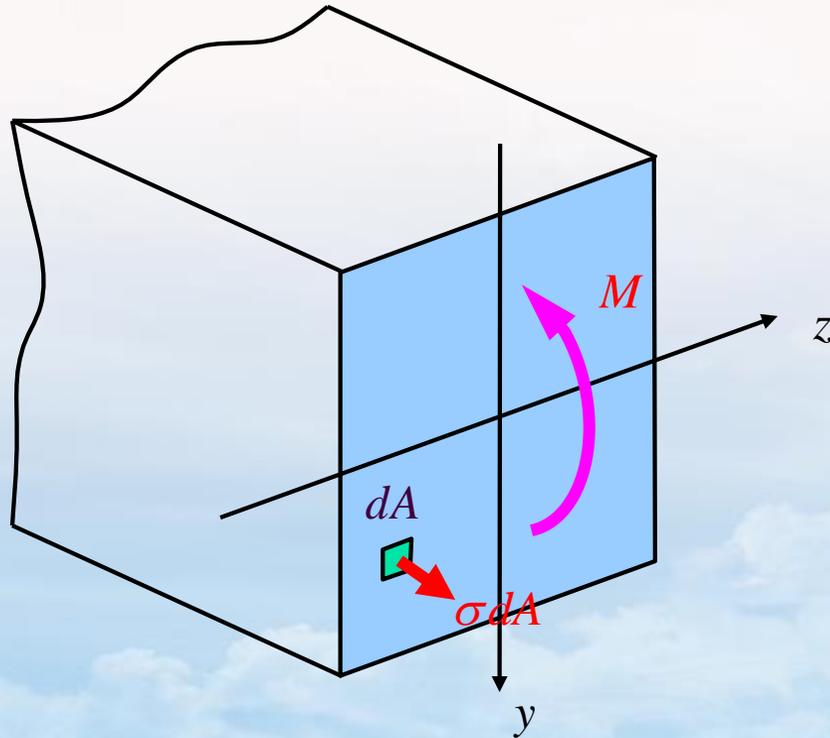
$$\sigma_{\max} = \frac{M}{J_z / y_{\max}}$$

$$\sigma_{\max} = \frac{M}{W_z} \quad (3)$$

W_z : 抗弯截面模量



梁的应力



$$\sigma_{\max} = \frac{M}{W_z} \quad \textcircled{3}$$

适用条件：

- 纯弯曲
- 当梁的 **长度** 与 **高度** 之比 **大于或等于5** 时

