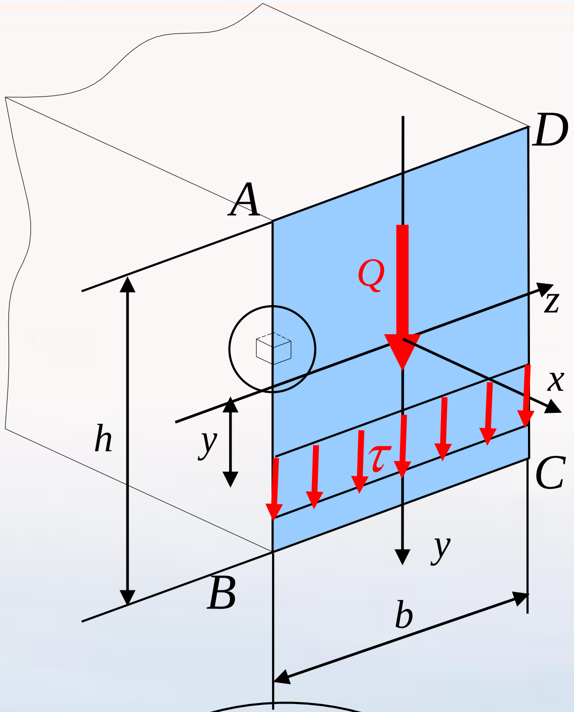
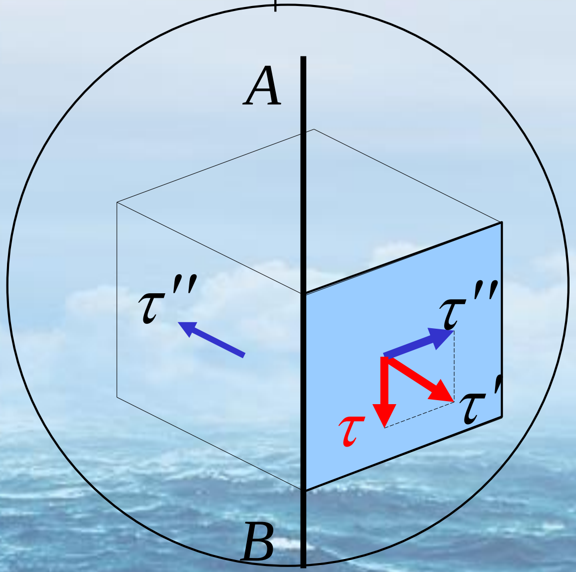
弯曲切应力

航海力学

梁的弯曲剪应力





两点假设

（1）在横截面上，任一点的切应力方向均与梁的侧边平行。对于矩形截面来说，梁的侧边与截面上的剪力平行。

（2）切应力沿截面宽度均匀分布，即距中性轴等远的点，切应力值也都相等。

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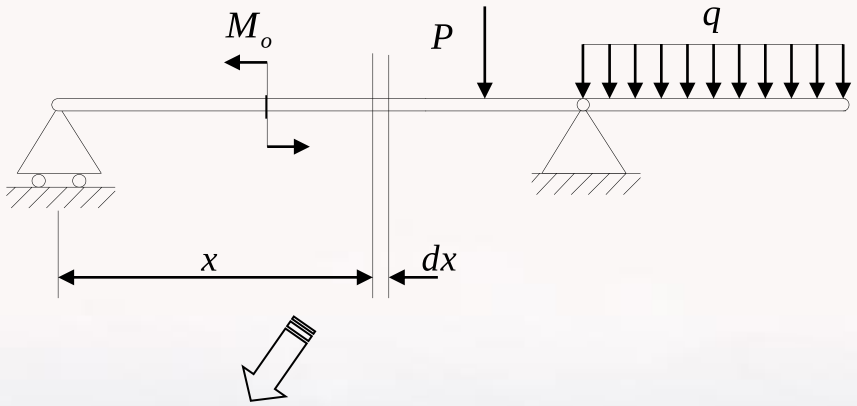

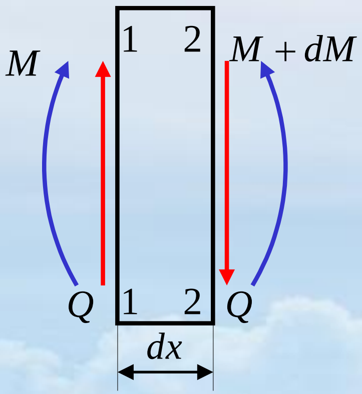

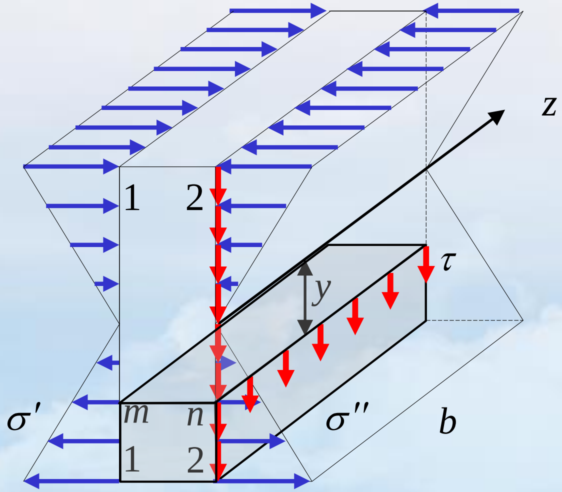

$$\sigma' = \frac{M(x)y^*}{J_z}$$

$$\sigma'' = \frac{[M(x)+dM(x)]y^*}{J_z}$$

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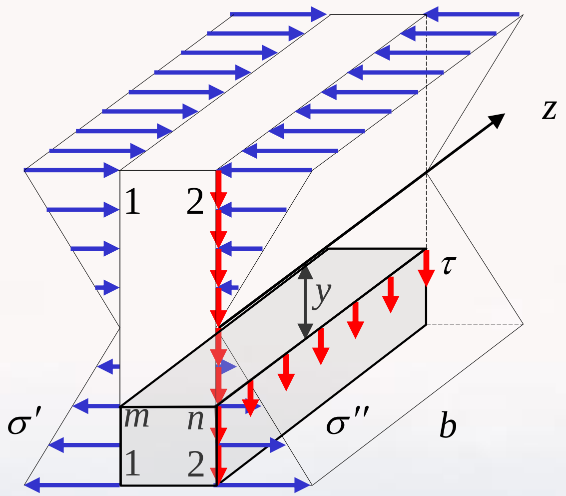

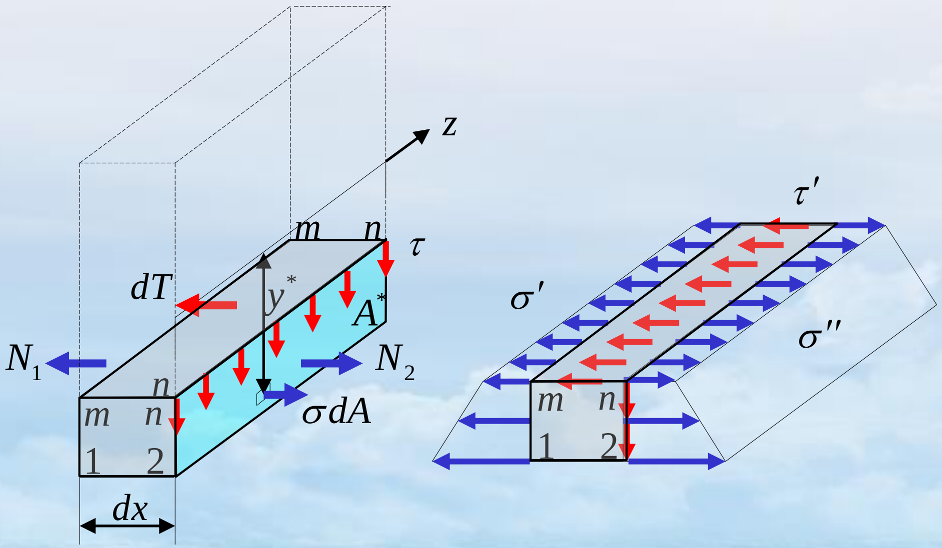

$$\Sigma F_x = 0, \qquad N_2 - N_1 - dT = 0$$

所以，$dT = N_2 - N_1$ (a)

$$dT = \tau' b dx \qquad (b)$$

$$N_1 = \int_{A^*} \sigma' dA$$

$$= \int_{A^*} \frac{M(x) y^*}{J_z} dA$$

$$= \frac{M(x)}{J_z} \int_{A^*} y^* dA$$

$$= \frac{M(x) S_z^*}{J_z}$$

$$N_1 = \frac{M(x) S_z^*}{J_z} \qquad (c)$$

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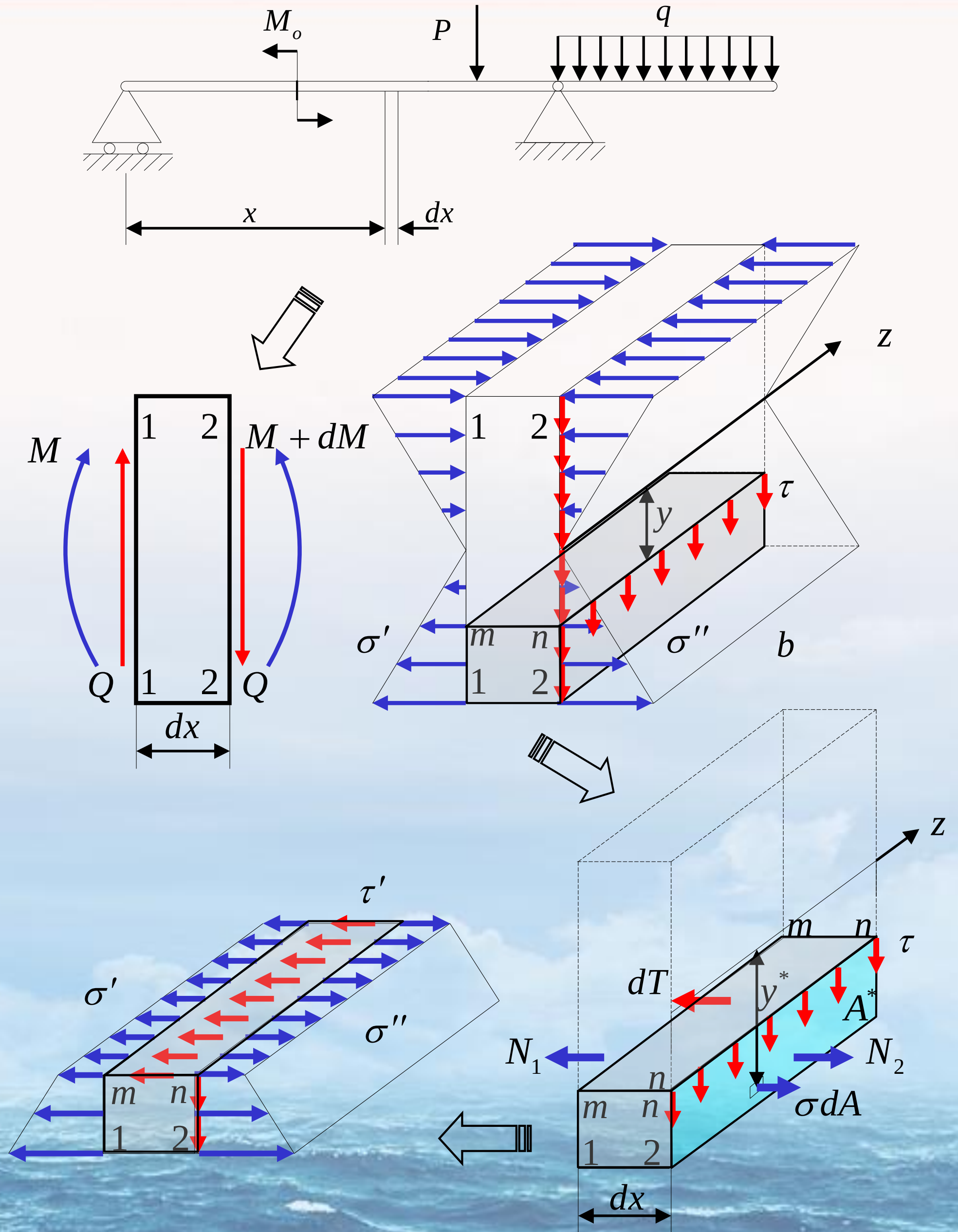

同理，

$$N_2=\frac{[M(x)+\mathrm{d}M(x)]S_z^*}{J_z}$$

由Q和M的微分关系可知，

$$dM(x)=Q(x)dx$$

所以，

$$N_2=\frac{[M(x)+Q(x)dx]S_z^*}{J_z} \qquad \text{(d)}$$

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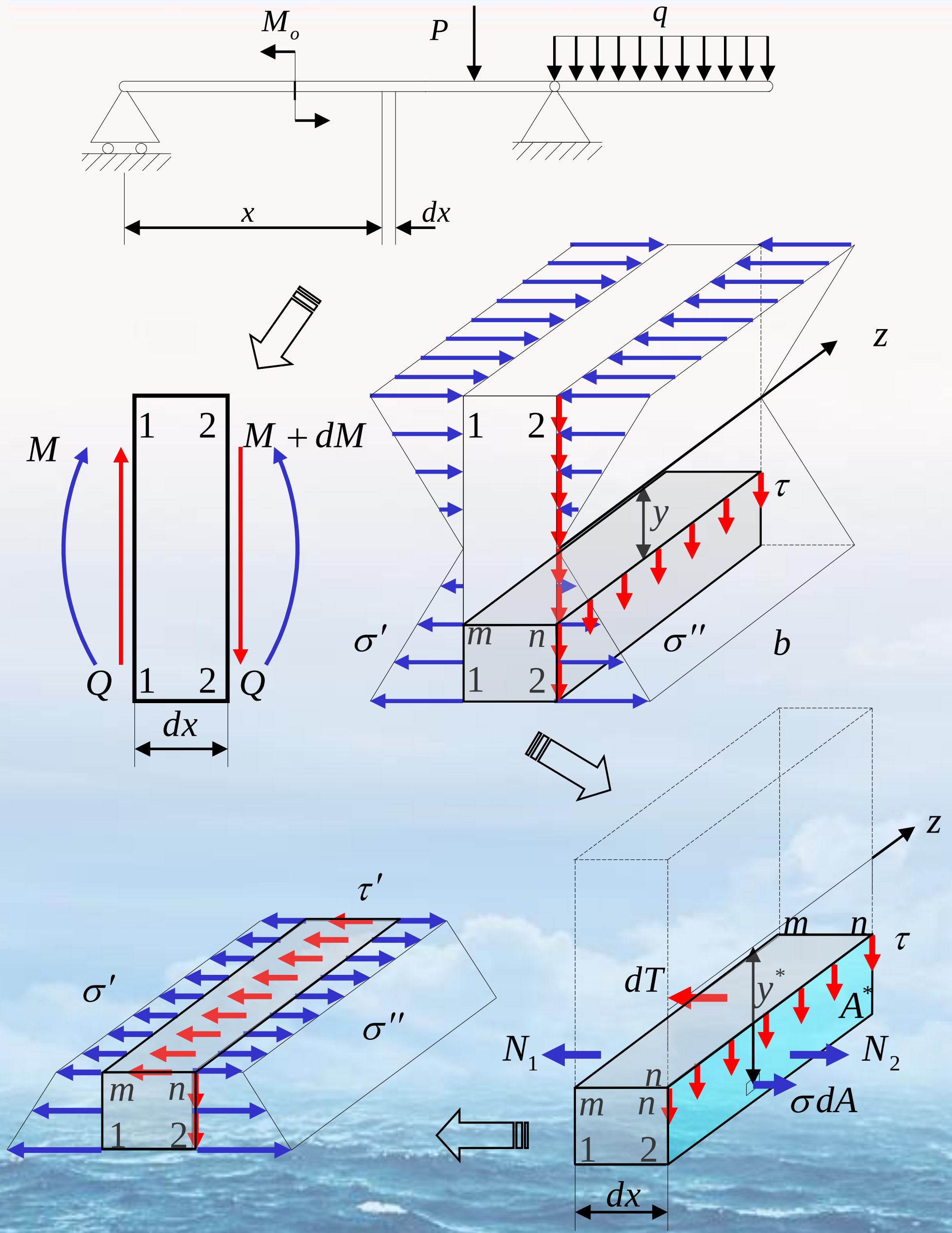

$$dT = \tau' b dx \qquad (b)$$

$$N_1 = \frac{M(x) S_z^*}{J_z} \qquad (c)$$

$$N_2 = \frac{[M(x)+Q(x)dx] S_z^*}{J_z} \qquad (d)$$

$$dT = N_2 - N_1 \qquad (a)$$

$$\tau' b dx = \frac{[M(x)+Q(x)dx] S_z^*}{J_z} - \frac{M(x) S_z^*}{J_z}$$

$$\tau' = \frac{Q(x)\ S_z^*}{J_z \cdot b}$$

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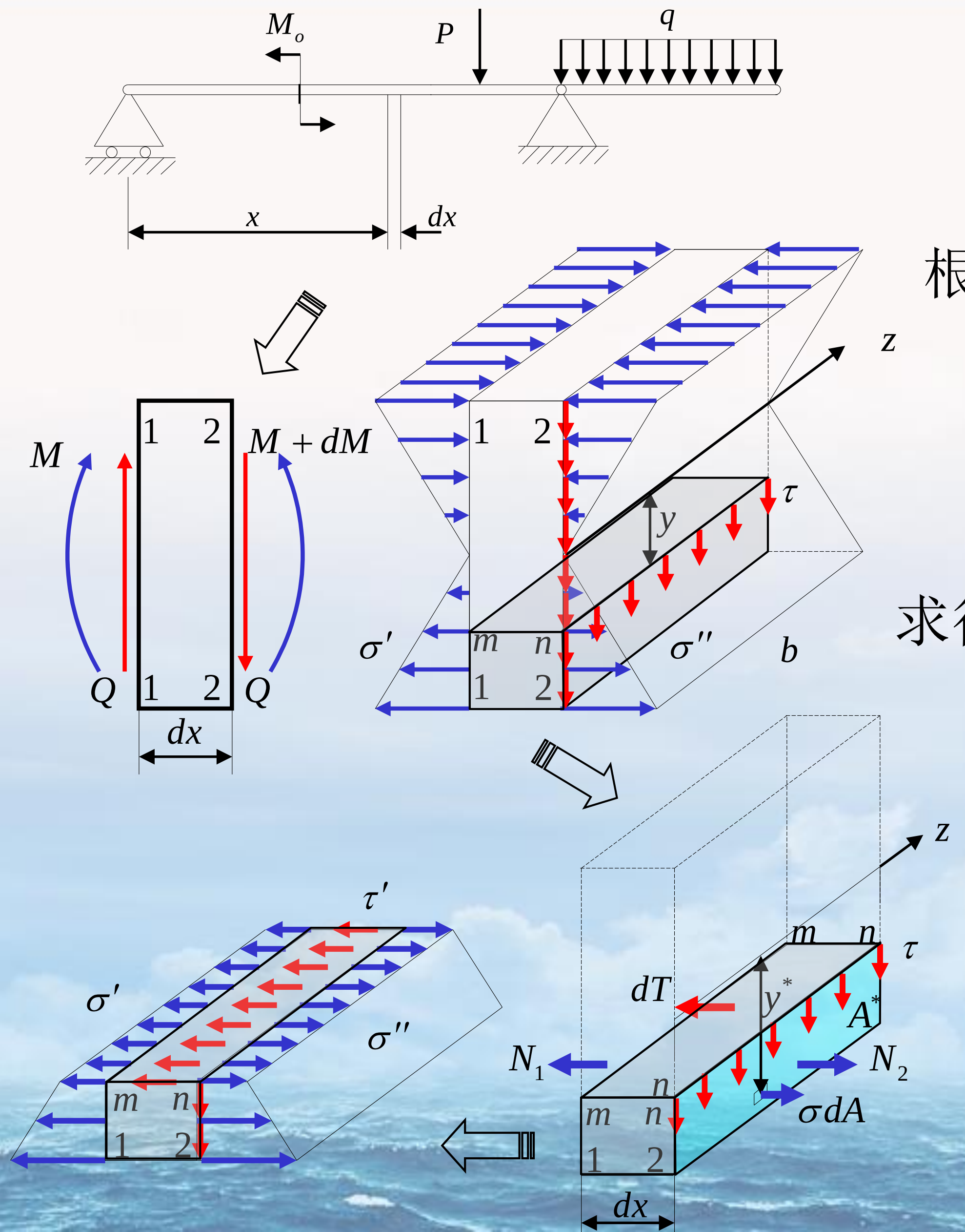

$$\tau' = \frac{Q(x)\ S_z^*}{J_z \cdot b}$$

根据切应力互等定理，

$$\tau(y)=\tau'$$

求得横界面距中性轴为y处的切应力为

$$\tau(y)=\frac{Q(x)S_z^*}{J_z \cdot b}$$

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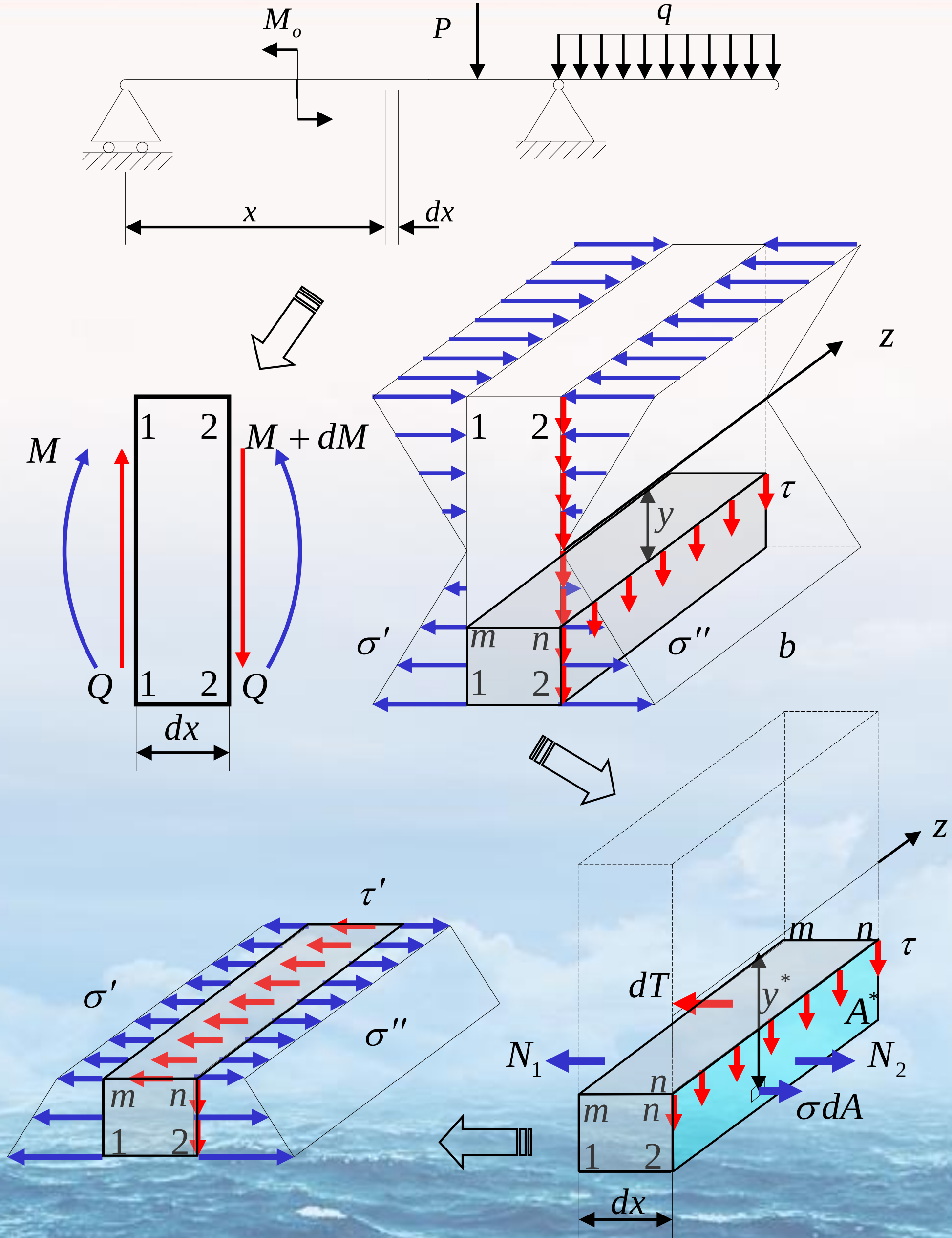

$$\tau(y)=\frac{Q(x)S_z^*}{J_z\cdot b}$$

对于某一确定界面，

$Q(x), J_z$和b均为常量

S_z^*为y的函数

$$S_z^*=A^*\cdot y_c^*$$

$$=b(\frac{h}{2}-y)\frac{1}{2}(\frac{h}{2}+y)$$

$$=\frac{b}{2}[(\frac{h}{2})^2-y^2]$$

$$=\frac{b}{2}[\frac{h^2}{4}-y^2]$$

$$S_z^*=\frac{b}{2}[\frac{h^2}{4}-y^2]$$

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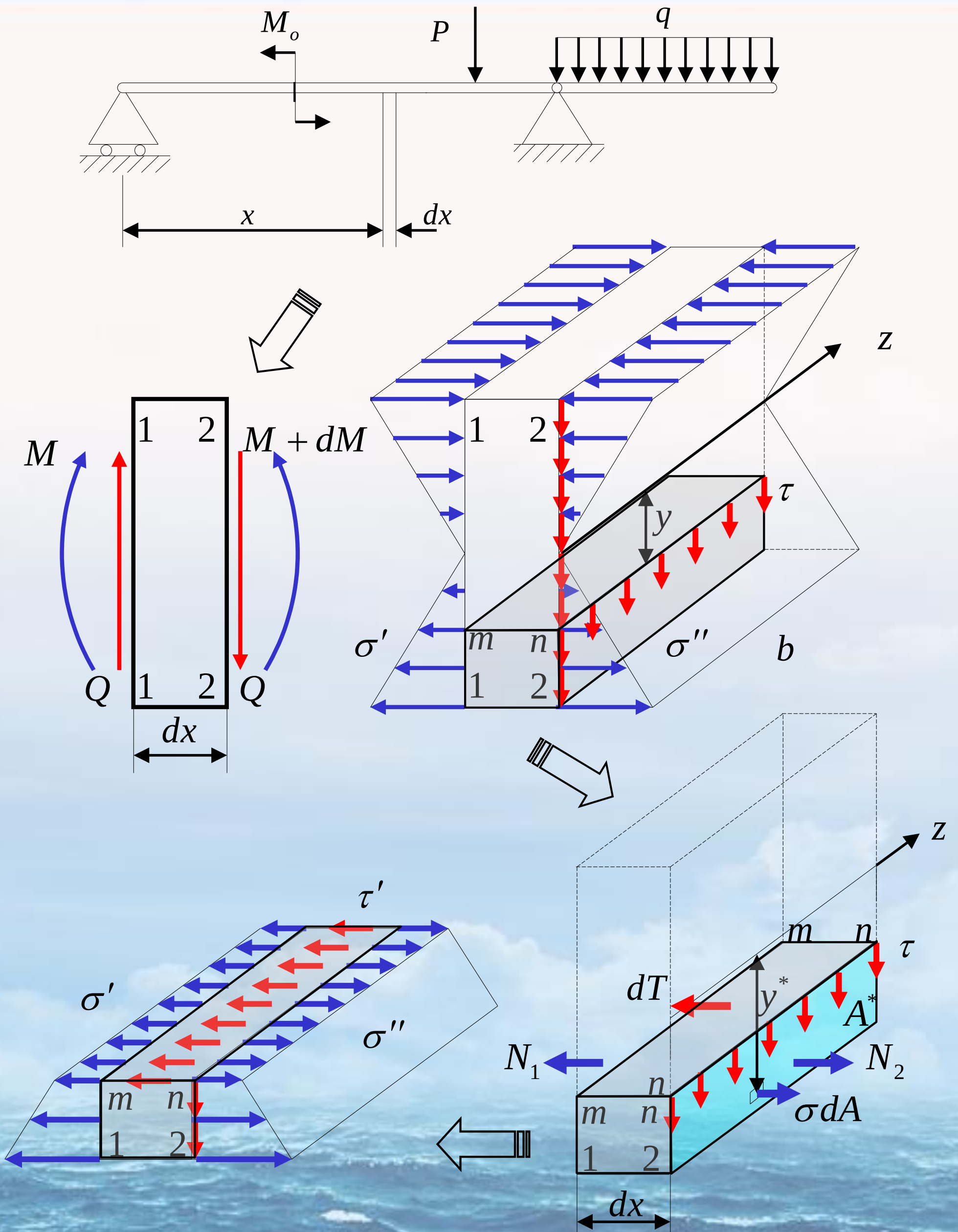

$$S_z^* = \frac{b}{2}\left[\frac{h^2}{4} - y^2\right]$$

$$\tau(y) = \frac{Q(x)S_z^*}{J_z \cdot b}$$

$$\tau(y) = \frac{Q(x)\dfrac{b}{2}\left[\dfrac{h^2}{4} - y^2\right]}{\dfrac{bh^3}{12}\cdot b}$$

$$= \frac{6Q(x)}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

所以，剪应力沿界面高度按二次抛物线分布。

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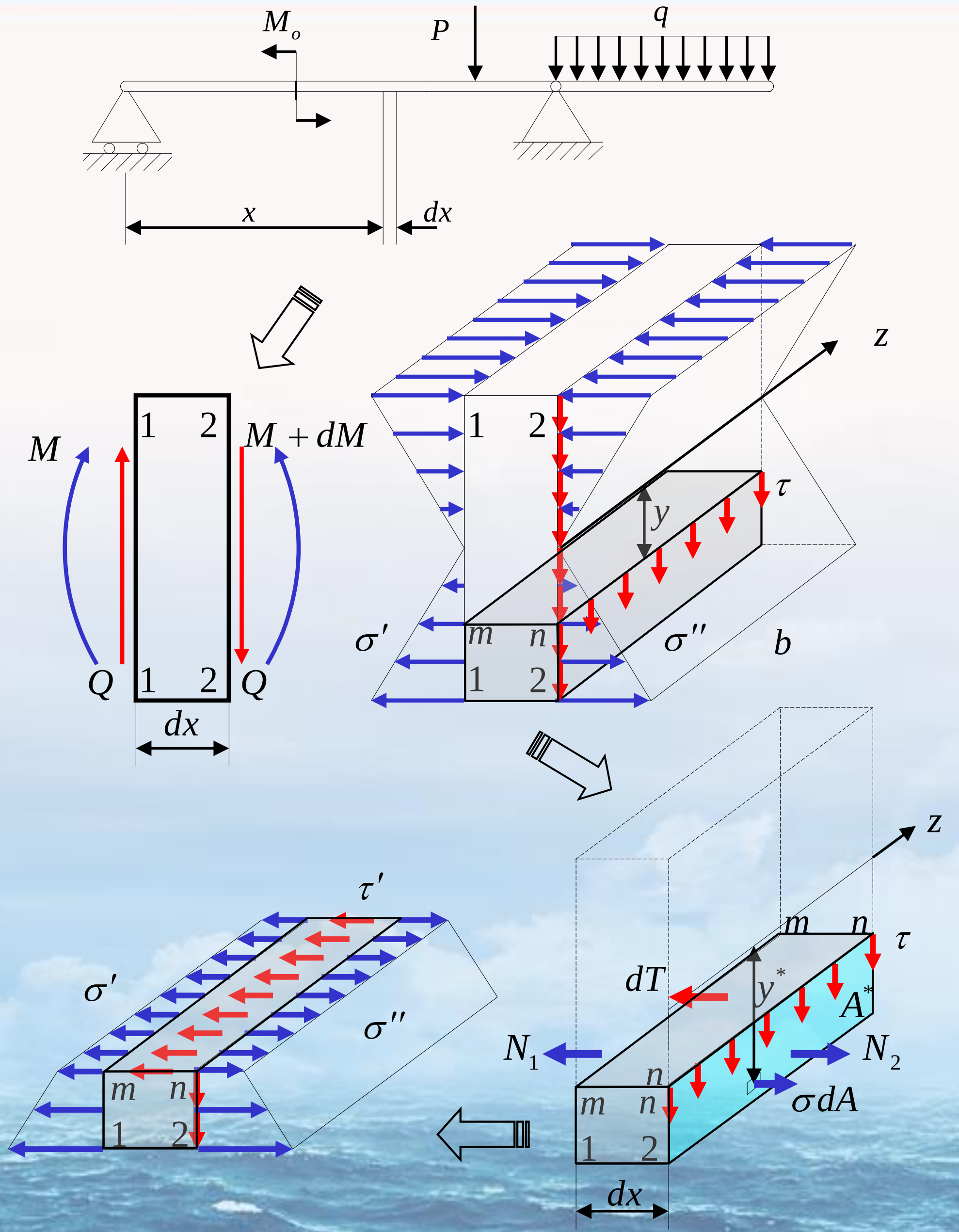


$$\tau(\mathrm{y})=\frac{6Q(x)}{bh^3}\left(\frac{h^2}{4}-y^2\right)$$

当$y=\pm\dfrac{h}{2}$时，

$$\tau=0$$

当$y=0$时，

$$\tau_{\max}=\frac{6Q(x)}{2bh}$$

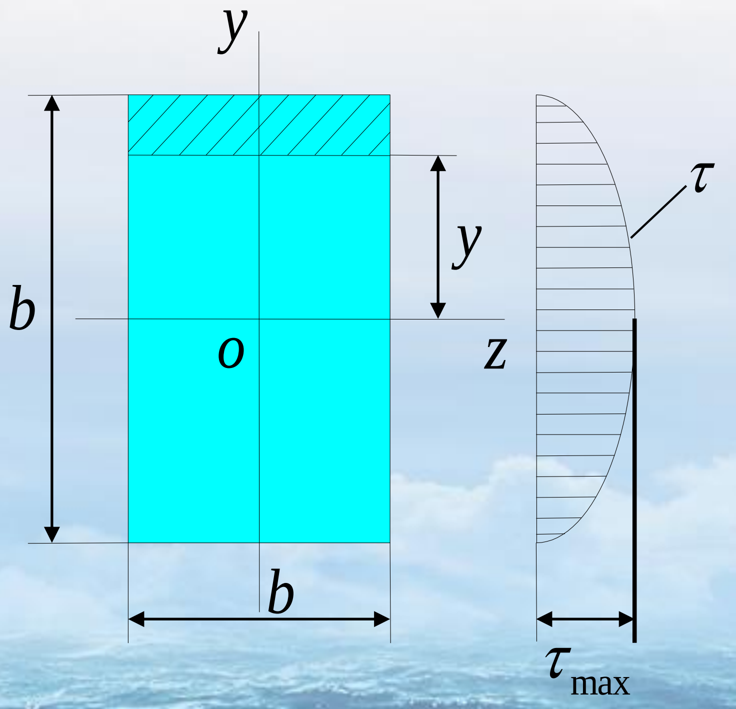
即，

$$\tau_{\max}=\frac{3}{2}\frac{\mathrm{Q}}{\mathrm{A}}$$

梁的应力&梁的剪应力

梁弯曲时，横截面上的剪应力的计算公式：

$$\tau = \frac{QS_z}{J_z b}$$



τ:横截面上离中性轴为y处的剪应力,MPa

Q:横截面上的剪力,N

b:所求剪应力处横截面的宽度,mm

J_z:整个横截面对中性轴的惯矩,mm^4

S_z:由所求剪应力处的宽度向外侧算得的面积对中性轴的静矩,mm^3

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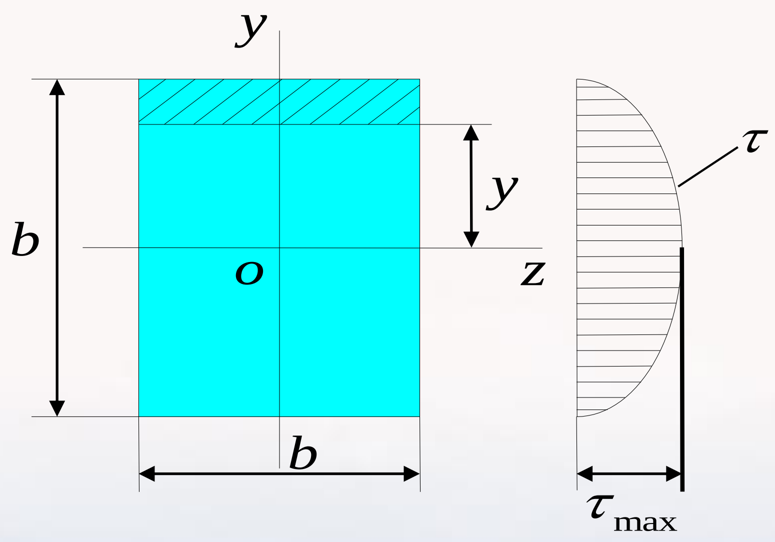
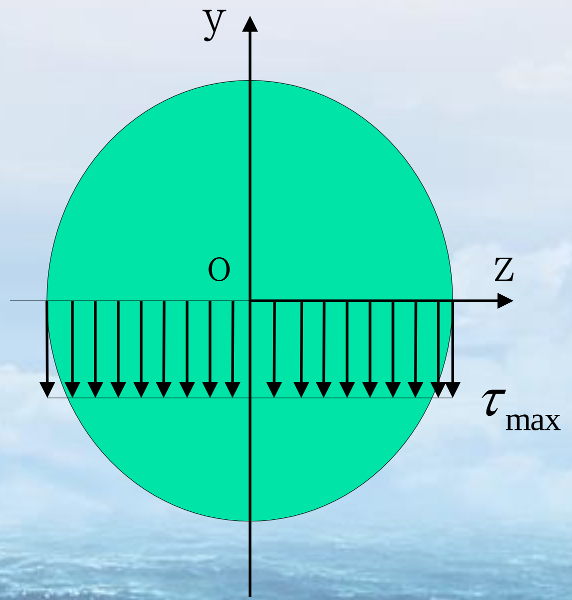


$$\tau_{\max} = \frac{3}{2}\frac{Q}{A}$$



$$\tau_{\max} = \frac{4}{3}\frac{Q}{A}$$