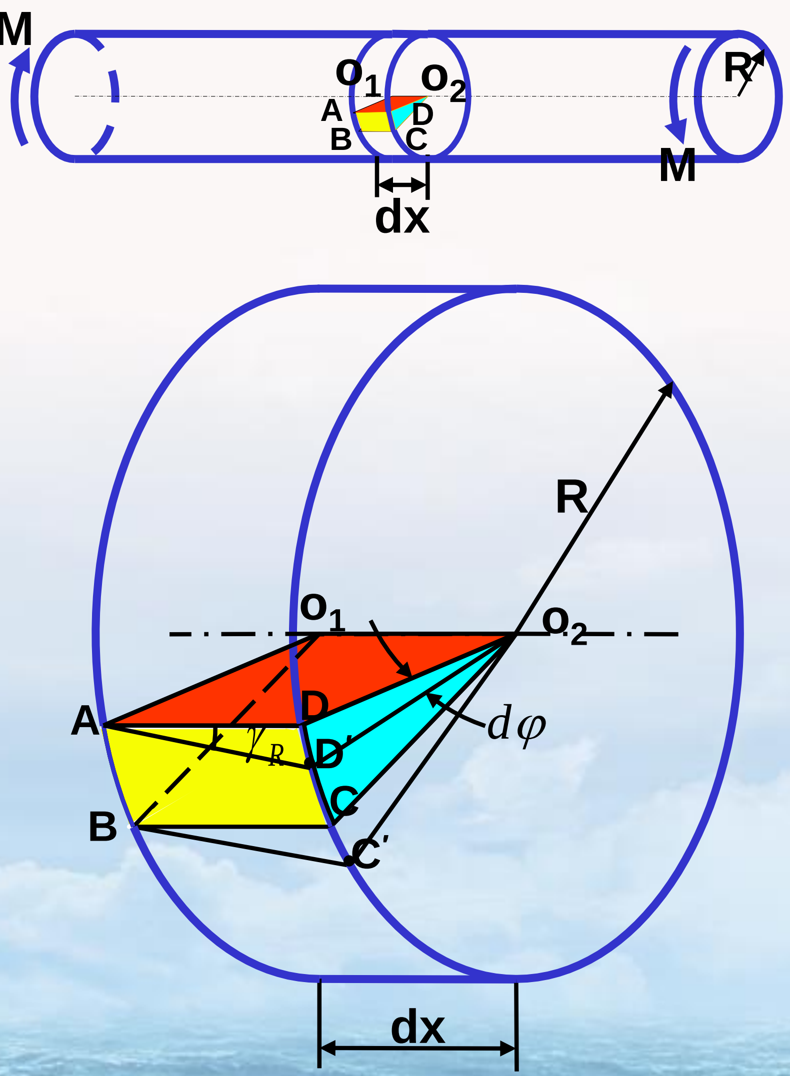
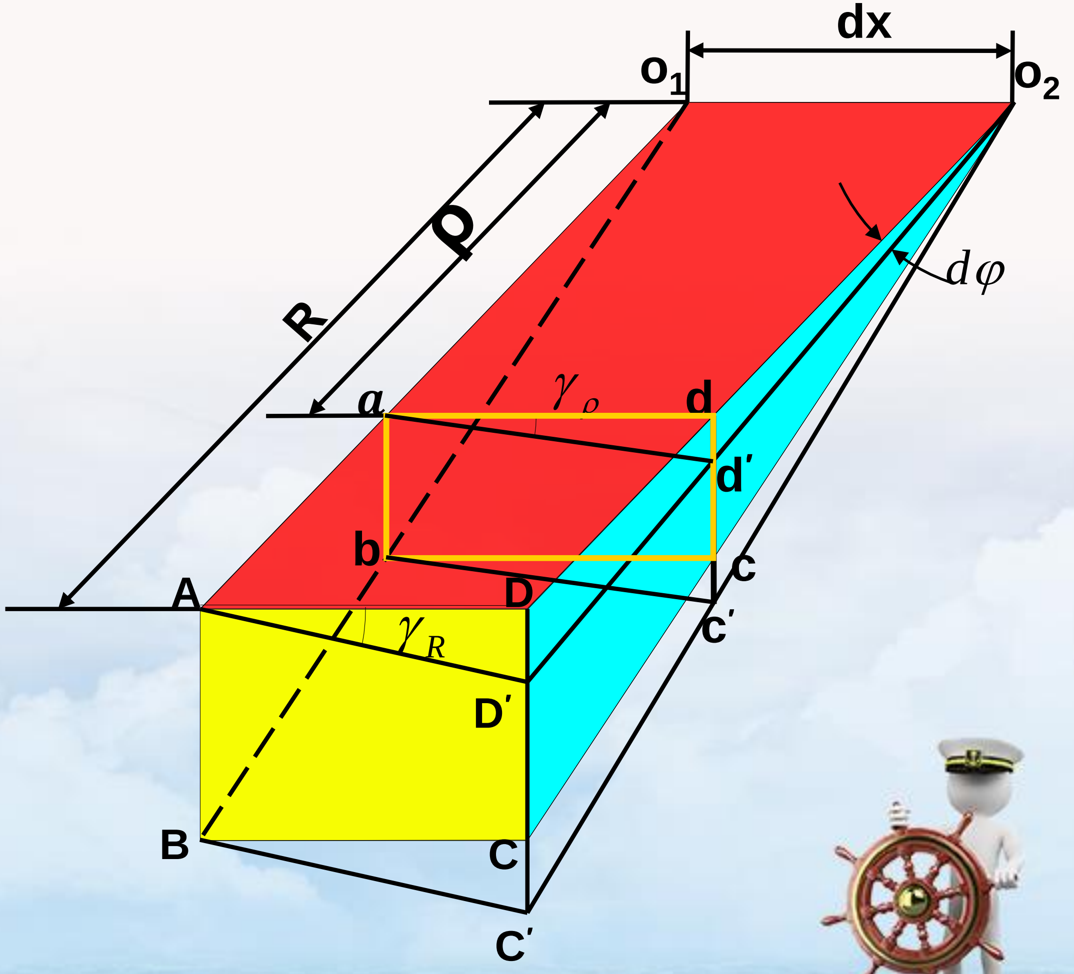
# 扭转应力

# 扭 转


M
O1
O2
R
A
B
D
C
M
dx
R
O1
O2
A
D
D′
B
C
C′
dφ
γR
dx


dx
O1
O2
R
ρ
a
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d′
b
c
c′
γρ
dφ
A
D
γR
D′
B
C
C′

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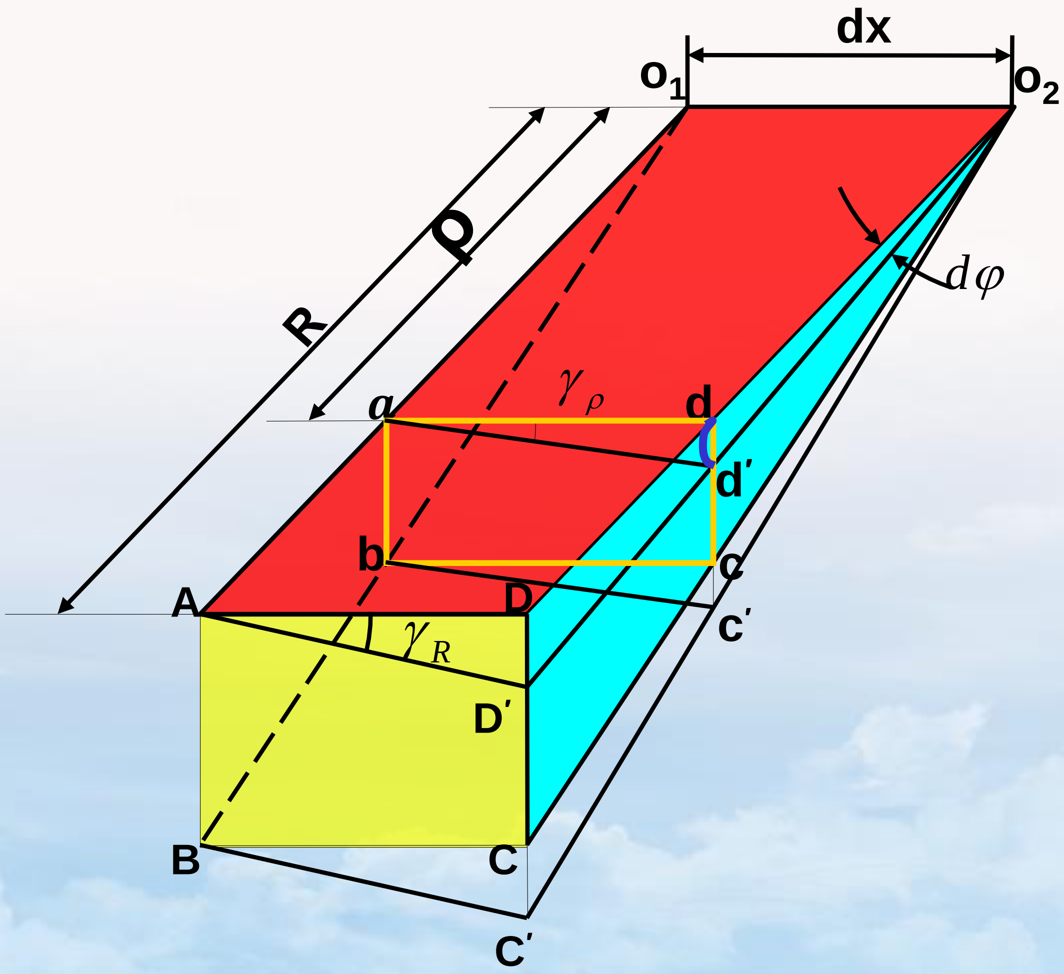



1、几何方面

$$\gamma_\rho \approx \tan\gamma_\rho = \frac{\overline{dd'}}{\overline{ad}} \approx \frac{dd'}{\overline{ad}} = \frac{\rho d\varphi}{dx}$$

$$\gamma_\rho = \rho\frac{d\varphi}{dx} \quad ①$$

2、物理方面

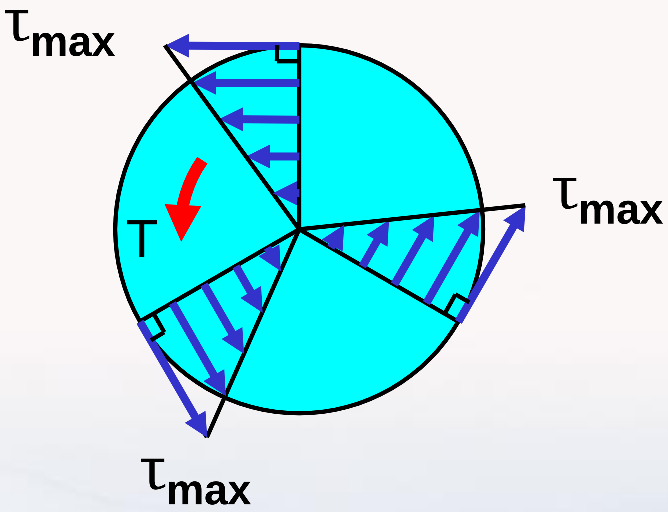
$$\tau_\rho = G\gamma_\rho = G\rho\frac{d\varphi}{dx}$$

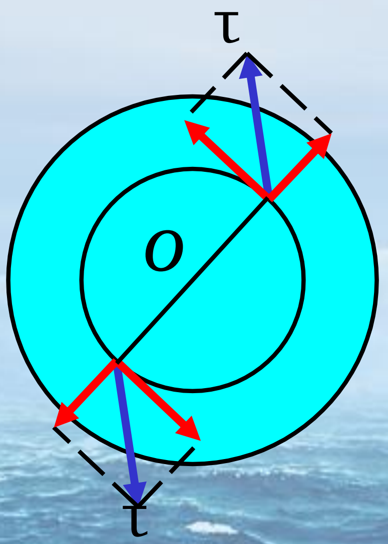
$$\tau_\rho = G\rho\frac{d\varphi}{dx} \quad ②$$

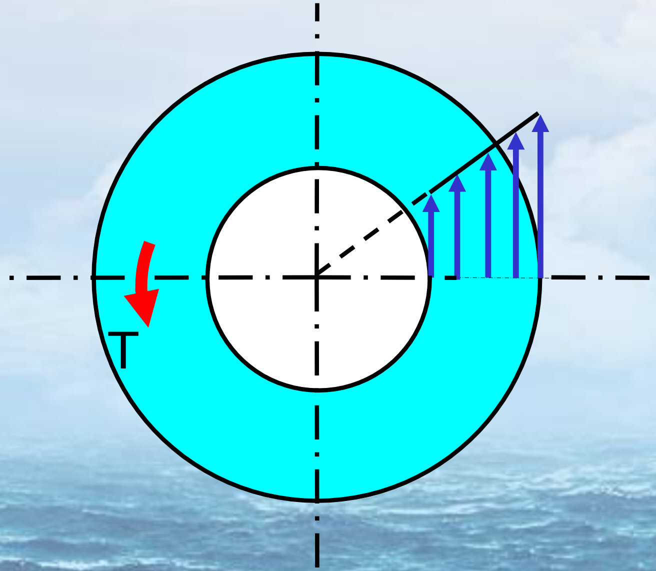
# 扭 转

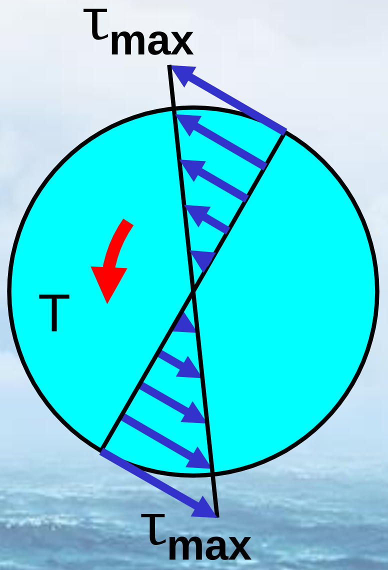
$$\tau_{\rho} = G\rho\frac{d\varphi}{dx} \quad ②$$

- 当ρ=0时，τ=0
- 当ρ=R时，$\tau=\tau_{max}$
- 当0<ρ<R时，τ沿半径呈线性分布。
- 切应力的方向与半径垂直。



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3、静力学方面

$$\tau_\rho = G\rho\frac{d\varphi}{dx}$$

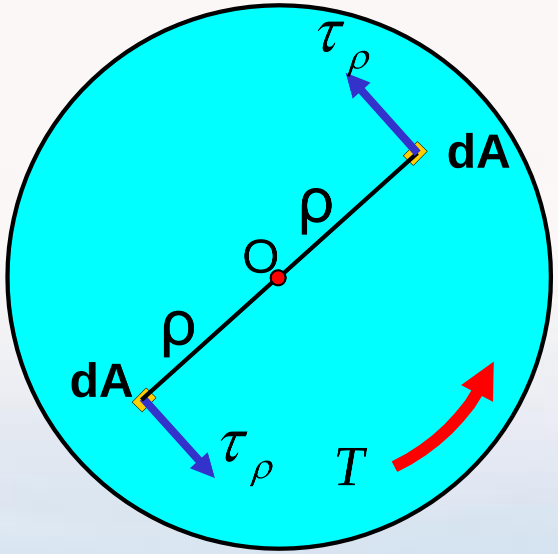
$$\int_A \rho\cdot\tau_\rho\cdot dA = T$$

$$\tau_\rho = G\rho\frac{d\varphi}{dx} \qquad ②$$

$$\int_A \rho G\rho\frac{d\varphi}{dx}dA = T$$

$$G\frac{d\varphi}{dx}\int_A \rho^2 dA = T$$

$J_p$：极惯矩




$$\frac{d\varphi}{dx} = \frac{T}{GJ_p} \qquad ③$$

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$$\frac{d\varphi}{dx}=\frac{T}{GJ_p} \quad ③$$

$$\tau_\rho = G\rho\frac{d\varphi}{dx} \quad ②$$

$$\tau_\rho = G\rho\frac{T}{GJ_p}=\frac{T}{J_p}\rho \longrightarrow \boxed{\tau_\rho=\frac{T}{J_p}\rho}$$

当$\rho = R$时，$\tau$取最大值$\tau_{\max}$：

$$\tau_{\max}=\frac{T}{(J_P/R)} \xrightarrow[\text{抗扭截面模量}]{W_p=(J_P/R)} \boxed{\tau_{\max}=\frac{T}{W_p}}$$

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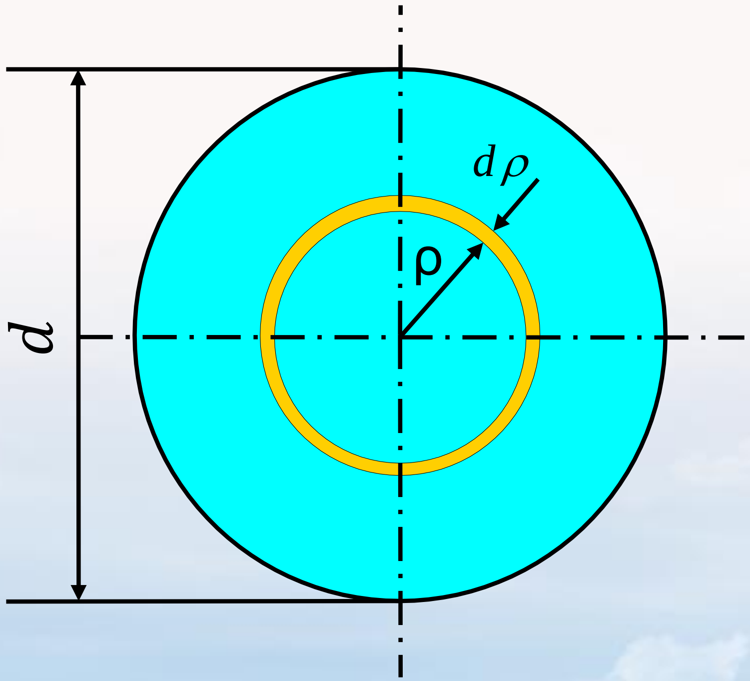
圆截面的极惯矩：




$$J_P = \frac{\pi d^4}{32}$$

$$W_P = \frac{\pi d^3}{16}$$

$$dA = 2\pi\rho d\rho$$

$$J_p = \int_A \rho^2 dA$$

$$= \int_0^{\frac{d}{2}} \rho^2 \cdot 2\pi\rho d\rho$$

$$= 2\pi \int_0^{\frac{d}{2}} \rho^3 d\rho$$

$$= [2\pi \cdot \frac{\rho^4}{4}]_0^{\frac{d}{2}}$$

$$= 2\pi \times \frac{1}{4} \times \frac{d^4}{16} = \frac{\pi}{32} d^4$$

$$W_p = \frac{J_p}{d/2} = \frac{\frac{\pi}{32} d^4}{d/2} = \frac{\pi d^3}{16}$$

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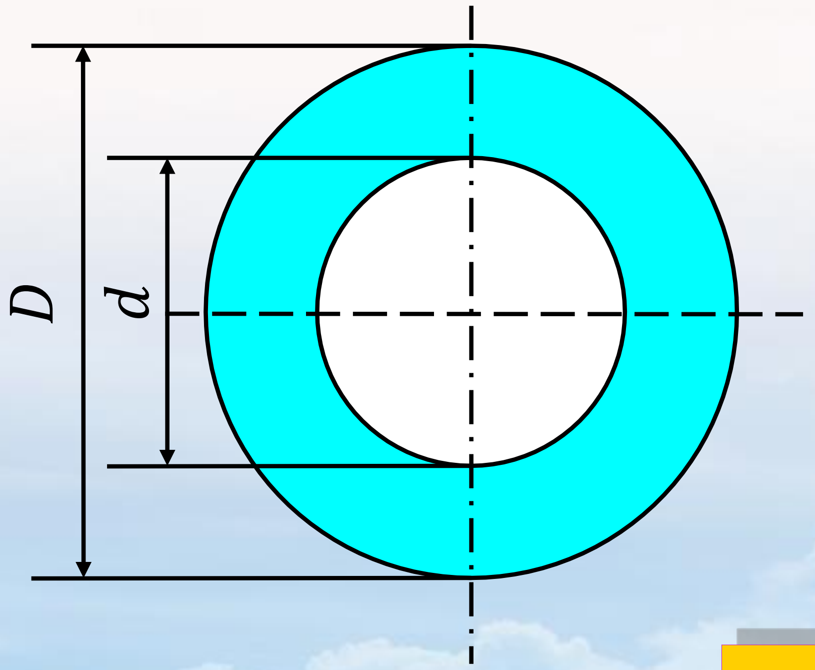
圆环截面的极惯：




$$J_p = \int_{\frac{d}{2}}^{\frac{D}{2}} \rho^2 dA$$

$$= \frac{\pi}{32}(D^4 - d^4)$$

令 $\alpha = \dfrac{d}{D}$

$$J_p = \frac{\pi}{32} D^4 (1-\alpha^4)$$

$$W_p = \frac{J_p}{D/2} = \frac{\frac{\pi}{32} D^4 (1-\alpha^4)}{D/2} = \frac{\pi D^3}{16}(1-\alpha^4)$$